# Nonlinear magnetic electron tripolar vortices in streaming plasmas

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Magnetic electron modes in nonuniform magnetized and unmagnetized streaming plasmas, with characteristic frequencies between the ion and electron plasma frequencies and at spatial scales of the order of the collisionless skin depth, are studied. Two coupled equations, for the perturbed (in the case of magnetized plasma) or self-generated (for the unmagnetized plasma case) magnetic field, and the temperature, are solved in the strongly nonlinear regime and stationary traveling solutions in the form of tripolar vortices are found.

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## I. INTRODUCTION

Magnetic electron modes are known to exist in inhomogeneous unmagnetized [1] and magnetized plasmas [2,3]. They appear on spatial scales of the order of the collisionless skin depth and at frequencies between the ion and electron plasma frequencies. In the problem of unmagnetized plasmas they lead to the spontaneous generation of a magnetic field [4], occurring as a first-order baroclinic effect, i.e., due to a term of the form $\vec{\nabla} n_0 \times \vec{\nabla} T_1$, where $n_0$ and $T_1$ are the nonuniform basic state concentration and the electron temperature perturbation, respectively. In the nonlinear limit, in some circumstances, these linearly unstable modes can saturate into dipolar vortex structures of localized magnetic field [5,6] and a vortex chain [7]. In the case of a magnetized plasma [2], in the linear limit, an oscillatory instability may arise, which in the nonlinear regime leads to the formation of a dipolar vortex. In the presence of free energy in the form of a shear flow perpendicular to the magnetic field lines, this instability may cause the generation of a vortex chain [8]. The inclusion of the ion dynamics [3] results in the density perturbation and in new classes of modes and instabilities.

Tripolar vortices, the subject of this paper, are relatively novel phenomena in plasma physics [9–12], although they are well known in theory, and also in experiments with rotating fluids [13], and they have been observed in the seas of our planet [14]. In principle, they consist of a rotating central vortex with two satellites of opposite vorticity. In experiments with fluids they develop from slightly disturbed monopolar vortices, around the axis of rotation of rotating vessels with a rigid body rotation profile; in our plasma problems they appear in the linear profile of the plasma flow and are carried in the direction of it. Being stationary and traveling, they have regions of closed streamlines and therefore can carry plasma particles efficiently, so they can be of importance from the plasma transport point of view. In fluids they are proven to be robust and long-living structures that survive many rotations of the system; the remarkable example reported in Ref. [14] was visible for a period of several months. Locally, in plasmas they represent a rather strong nonuniformity in the system, and the effects of linear wave trapping and scattering, similar to the theory developed for the interaction of linear drift waves with dipolar vortices [15,16], should be worth investigating.

In this paper, starting with the standard set of equations describing electron magnetic modes in inhomogeneous magnetized and unmagnetized plasmas, we derive basic nonlinear equations for the perturbed (and self-generated) magnetic field and for the temperature, which in some regimes are known to possess nonlinear solutions in the form of a vortex chain [7,8]. Here, we present nonlinear solutions driven by vector-product type nonlinearities, i.e., tripolar stationary vortex structures settled in the plasma flow and carried in the direction of it, which resemble structures discovered in experiments with rotating fluids and obtained in some plasma problems, that is, on quite different time and spatial scales.

## II. MODEL AND DERIVATIONS

The model of a weakly inhomogeneous electron-ion plasma is used, with perturbations satisfying the condition $|k^{-1}| \ll L_n, L_T$, where $k$ is the wave number of perturbations existing in the system, and $L_n, L_T$ are the characteristic lengths of the concentration and temperature inhomogeneities, respectively. Perturbations of high frequency compared to the ion plasma frequency, i.e., $\omega_{pi} \ll \partial/\partial t \ll (\omega_{pe}, c\vec{\nabla})$, are studied, so that the heavy ions make a neutralizing background, and we study slow electron motion, neglecting the displacement current. The density perturbations are neglected, i.e., $n \equiv n_0(x)$, there is a plasma flow in the basic state along the $y$ axis, $\vec{v}_0 = v_0(x)\vec{e}_y$, and all basic state quantities are $x$-dependent functions.

The basic equations describing electron magnetic modes in such a system are, respectively, the electron momentum and energy equations, and the Maxwell equations:

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}\right)\vec{v} = -\frac{e}{m}(\vec{E} + \vec{v}\times\vec{B}) - \frac{1}{mn}\vec{\nabla}(nT), \quad (2.1)$$

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}\right)(Tn^{1-\gamma}) = 0, \quad (2.2)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}, \quad (2.3)$$

$$\vec{\nabla}\times\vec{B} = -\mu_0 e n \vec{v}. \quad (2.4)$$

 

The above set of equations will be used to investigate two different cases, i.e., perturbations in magnetized and unmagnetized plasma, and flutelike, $z$-independent perturbations will be studied.

### A. Magnetized plasma

Assume a spatially nonuniform magnetic field $\vec{B}_0 = B_0(x)\vec{e}_z$ and the plasma concentration $n_0(x)$, causing an electron flow $v_0(x)\vec{e}_y$ in the basic state, perpendicular to both the magnetic field lines and the basic state gradients. Using Eqs. (2.1)–(2.4) one can find that the stationary basic state is described by

$$\frac{d}{dx}\left(\frac{B_0^2}{2\mu_0}+n_0T_0\right)=0, \tag{2.5}$$

and

$$\vec{v}_0(x)=-\frac{1}{\mu_0 e n_0}\vec{\nabla}\times\vec{B}_0=\frac{c^2}{\omega_{pe}^2}\vec{e}_z\times\vec{\nabla}\Omega_0, \tag{2.6}$$

where $\Omega_0=eB_0/m$ and $\omega_{pe}$ are the electron gyrofrequency and plasma frequency, respectively, and the subscript 0 denotes the basic state quantities.

The following set of nonlinear equations for the perturbation of the magnetic field $B_1$ and electron temperature $T_1$ is obtained:

$$\frac{\partial}{\partial t}\left(\frac{1}{\mu_0 e n_0}\nabla^2-\frac{e}{m}-\frac{n_0'}{\mu_0 e n_0^2}\frac{\partial}{\partial x}\right)B_1+\left[\frac{1}{\mu_0 m}\left(\frac{B_0}{n_0}\right)' -\frac{e v_0}{m}-\frac{1}{\mu_0 e n_0}\left(v_0''-\frac{n_0' v_0'}{n_0}\right)\right]\frac{\partial B_1}{\partial y}-\frac{n_0'}{m n_0}\frac{\partial T_1}{\partial y}+\frac{v_0}{\mu_0 e n_0}\frac{\partial}{\partial y}\nabla^2 B_1-\frac{v_0 n_0'}{\mu_0 e n_0^2}\frac{\partial^2 B_1}{\partial x\partial y}+\frac{1}{(\mu_0 e n_0)^2}\{B_1,\nabla^2 B_1\}=0, \tag{2.7}$$

$$\frac{\partial T_1}{\partial t}+\frac{1}{\mu_0 e n_0}\{B_1,T_0+T_1\}+v_0(x)\times\frac{\partial T_1}{\partial y}+\frac{(\gamma-1)T_0}{\mu_0 e n_0^2}\{n_0,B_1\}=0. \tag{2.8}$$

Here the Poisson bracket notation is used,

$$\{f_1,f_2\}\equiv\frac{\partial f_1}{\partial x}\frac{\partial f_2}{\partial y}-\frac{\partial f_1}{\partial y}\frac{\partial f_2}{\partial x},$$

and

$$\vec{\nabla}\equiv\frac{\partial}{\partial x}\vec{e}_x+\frac{\partial}{\partial y}\vec{e}_y,$$

the primes denote the $x$ derivatives of the corresponding basic state functions, and other notation is standard.

The linearized Eqs. (2.7) and (2.8), in the local approach when the basic state gradients can be assumed as constants, yield an oscillatory instability [8], which is closely connected with the direction of the basic state gradients $n_0'$ and $T_0'$.

For perturbations with typical wavelengths of the order of the collisionless skin depth $\lambda_S=c/\omega_{pe}$, and with conditions $(\lambda_S,L_B)/L_n\ll 1$, we introduce the following notations and normalization:

$$v_0(x)=\widehat{v_0}f(x),\quad \Omega_1=\frac{eB_1}{m},\quad \frac{\lambda_S}{\widehat{v_0}}\frac{\partial}{\partial t}\rightarrow\frac{\partial}{\partial t},\quad \frac{(\Omega_0,\Omega_1)}{\widehat{v_0}/\lambda_S}\rightarrow(\Omega_0,\Omega_1),\quad \omega_{pe}^2=\frac{e^2 n_{00}}{m\varepsilon_0}. \tag{2.9}$$

Then with accuracy to the second-order small terms Eqs. (2.7) and (2.8) can be written as

$$\frac{\partial}{\partial t}(\nabla^2-1)\Omega_1-f''(x)\frac{\partial\Omega_1}{\partial y}-n_0'(x)\frac{\partial T_1}{\partial y}+f(x)\times\frac{\partial\nabla^2\Omega_1}{\partial y}+\left(\frac{\partial\Omega_1}{\partial x}\frac{\partial}{\partial y}-\frac{\partial\Omega_1}{\partial y}\frac{\partial}{\partial x}\right)\nabla^2\Omega_1=0, \tag{2.10}$$

$$\left(\frac{\partial}{\partial t}+\vec{e}_z\times\vec{\nabla}(\Omega_1+\Omega_0)\cdot\vec{\nabla}\right)[T_1-\psi(x)]=0. \tag{2.11}$$

Here, the dimensionalless function $\psi(x)=(\gamma-1)n_0(x)-T_0(x)$ is introduced, and $n_0(x)$ and $T_0(x)$ are normalized to some characteristic density and temperature of the system, $n_{00}$ and $T_{00}$, respectively, and, according to the basic state Eqs. (2.5) and (2.6), $f(x)=\Omega_0'(x)$.

In the nonlinear limit, looking for localized solutions that are stationary in the reference frame moving with constant velocity $u$ in the direction perpendicular to both the basic state gradients and the magnetic field lines, Eq. (2.11) can be integrated, giving

$$T_1-\psi(x)=\mathcal{F}(\Omega_1+\Omega_0-ux). \tag{2.12}$$

Here, $\mathcal{F}(\xi)$ is an arbitrary function of the given argument, and we may take it as linear, $\mathcal{F}(\xi)=F_1(\Omega_1+\Omega_0-ux)$, where $F_1$ is a constant. On the condition of vanishing perturbations at infinity we have

$$T_0'-(\gamma-1)n_0'=F_1(\Omega_0'-u) \tag{2.13}$$

and

$$T_1=F_1\Omega_1. \tag{2.14}$$

This, together with Eq. (2.5), yields the following connection between the basic state functions:

$$\Omega_0(x)=F_1\frac{(n_0T_0)'}{(\gamma-1)n_0'-T_0'-F_1u}. \tag{2.15}$$

Putting Eq. (2.14) into Eq. (2.10) we obtain

$$\left(\frac{\partial}{\partial t}+\Omega_0'\frac{\partial}{\partial y}+\vec{e}_z\times\vec{\nabla}\Omega_1\vec{\nabla}\right)(\nabla^2-1)\Omega_1 + (\Omega_0'-\Omega'''-F_1n_0')\frac{\partial\Omega_1}{\partial y}=0. \tag{2.16}$$

It is interesting to note that for $\Omega_0'=F_1n_0'$, Eq. (2.16) becomes identical to Eq. (13) from Ref. [7], obtained in a study of the formation of nonlinear vortex chains in a homogeneous plasma. As shown in a detailed analysis performed in that paper, in the case of a tanh($x$) profile of the flow, i.e., for $\Omega_0(x)=ux+A\log\cosh(\kappa x)$, where $A$ and $\kappa$ are some constants, the only unstable linear modes, occurring as a result of the Cherenkov interaction with the nonuniform flow, are those with wave numbers satisfying the condition $k>(\kappa^2-1)^{1/2}$. In the nonlinear limit these unstable modes can saturate into the stationary traveling, single and double vortex chains obtained in Ref. [7].

However, in the present study with the density gradient effects included, and for some specific profiles of the functions describing the basic state, a type of vortex solution in the form of a tripolar vortex will be found. In order to find it we proceed by integrating Eq. (2.16), yielding

$$(\nabla^2-1)\Omega_1-\Omega_0(x)+\Omega_0''(x)+F_1n_0(x)=\mathcal{G}(\Omega_1+\Omega_0-ux), \tag{2.17}$$

where $\mathcal{G}(\xi)$ is an arbitrary function, which will be taken in the form $\mathcal{G}(\xi)=G_0+G_1\xi$. Let the basic state be described by the following set of equations:

$$\Omega_0-ux=ax^2,\quad \text{which gives}\quad f(x)=u+2ax, \tag{2.18}$$

and

$$-\Omega_0(x)+\Omega_0''(x)+F_1n_0(x)=bx^2, \tag{2.19}$$

where $a$ and $b$ are some constants modeling the basic state, and giving a linear shear flow profile. In this case, using standard procedure [10–12], Eq. (2.17) can be solved in cylindrical coordinates, independently inside and outside a circle with the radius $r_0$, allowing for different values of the given constants $G_{0,1}$ for these two regions. On condition of localized solutions for $r\rightarrow\infty$ we find that

$$\Omega_1^+(r,\theta)=\beta_0K_0(\lambda_1r)+\beta_2K_2(\lambda_1r)\cos 2\theta, \tag{2.20}$$

where $K_{0,1}$ are the modified Bessel functions of the given order, the superscript + denotes the outside values of the given quantities, with respect to the given circle, and

$$G_1^+=\frac{b}{a},\quad G_0^+=0,\quad \lambda_1^2=1+G_1^+.$$

Similarly, the inside solution can be written in the form

$$\Omega_1^-(r,\theta)=\alpha_0J_0(\lambda_2r)-\frac{Ar^2}{2}-B + \left(\alpha_2J_2(\lambda_2r)-\frac{Ar^2}{2}\right)\cos 2\theta, \tag{2.21}$$

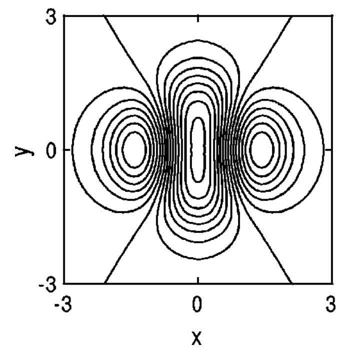


FIG. 1. Sketch of the contours of the magnetic tripole in magnetized plasma defined by Eqs. (2.20) and (2.21). Spatial scales are in units of $\lambda_S$. The two lateral vortices have the opposite sign from the central vortex.

where $J_{0,2}$ are the Bessel functions, and

$$1+G_1^-=-\lambda_2^2,\quad A=-\frac{b-G_1^-a}{1+G_1^-},$$

$$B=\frac{G_0^-}{1+G_1^-}-\frac{2(b-G_1^-a)}{(1+G_1^-)^2}.$$

The unknown constants in the solutions (2.20) and (2.21) can be found from the boundary conditions on the given circle, i.e., from the continuity conditions for the perturbation $\Omega_1$ and its first and second derivatives with respect to the coordinate $r$, assuming also that the argument $\xi$ is constant on the circle. A sketch of a typical tripolar vortex defined by Eqs. (2.20) and (2.21) is given in Fig. 1. It is nicely localized and, as follows from Eq. (2.18), it is created in the plasma flow at the position where the flow amplitude is equal to $u$, and carried by the flow in the direction that is perpendicular to the basic state gradients.

### B. Unmagnetized plasma

In the case of an unmagnetized plasma, Eqs. (2.1)–(2.4) in the linear domain, due to the first-order baroclinic term, describe the self-generation of the magnetic field [1], and are of importance in problems of laser-fusion plasmas. The inclusion of the ion dynamics may introduce new classes of instabilities, but they appear at much smaller time scales [3] and are not of importance for the present study.

The stationary basic state is trivially satisfied by the plasma flow $\vec{v}_0(x)=v_0(x)\vec{e}_y$, and

$$\frac{T_0'}{T_0}=-\frac{n_0'}{n_0}. \tag{2.22}$$

For the time and space scales used in the previous problem, we obtain the same equation for the temperature perturbation, i.e., Eq. (2.8), and a similar one for the self-generated magnetic field:

$$\frac{1}{\mu_0 e n_0}\frac{\partial}{\partial t}\nabla_\perp^2 B_1+\frac{1}{(\mu_0 e n_0)^2}\{B_1,\nabla_\perp^2 B_1\}=\frac{e}{m}\frac{\partial B_1}{\partial t}+\frac{e v_0}{m}\frac{\partial B_1}{\partial y}+\frac{1}{m n_0}\{n_0,T_1\}+\frac{1}{\mu_0 e n_0}\left(v_0''\frac{\partial B_1}{\partial y}-v_0\frac{\partial}{\partial y}\nabla_\perp^2 B_1\right). \quad (2.23)$$

In the local analysis Eqs. (2.8) and (2.23) yield a linear instability [7], which in the absence of the flow becomes a purely growing one. In the nonlocal linear case, assuming perturbations of the form $(T_1,\Omega_1)=[\hat{T}_1(x),\hat{\Omega}_1(x)]\exp(-i\omega t+iky)$, from Eqs. (2.8) and (2.23), and using Eq. (2.22), we obtain the following eigenvalue problem equation for $\hat{\Omega}_1(x)$:

$$\hat{\Omega}_1''(x)-(k^2+1)\hat{\Omega}_1(x)+\frac{\epsilon(x)\alpha(x)+f''(x)\beta(x)\delta}{\alpha^2(x)+\beta^2(x)}k\hat{\Omega}_1(x)+i\frac{f''(x)\alpha(x)\delta-\epsilon(x)\beta(x)}{\alpha^2(x)+\beta^2(x)}k\hat{\Omega}_1(x)=0, \quad (2.24)$$

where we introduced the real and imaginary parts of the frequency $\omega=\omega_r+i\delta$, and

$$\epsilon(x)=-k\gamma T_0(x)\left(\frac{n_0'(x)}{n_0}\right)^2+f''(x)[\omega_r-f(x)k],$$

$$\alpha(x)=[\omega_r-f(x)k]^2-\delta^2,$$

$$\beta(x)=2\delta[\omega_r-f(x)k].$$

An instability of the Kelvin-Helmholtz type can be readily found if at any position across the flow the following condition is satisfied:

$$f''(x)=2k\gamma T_0(x)\left(\frac{n_0'(x)}{n_0(x)}\right)^2\frac{\omega_r-f(x)k}{[\omega_r-f(x)k]^2+\delta^2}. \quad (2.25)$$

In the nonlinear limit the temperature equation becomes

$$\left(\frac{\partial}{\partial t}+\vec{e}_z\times\vec{\nabla}(\Omega_1+\varphi)\cdot\vec{\nabla}\right)[T_1-\psi(x)]=0, \quad (2.26)$$

where $\varphi'(x)=f(x)$, and $\psi(x)$ was defined earlier. As in the previous case Eq. (2.26) can be integrated, yielding Eq. (2.14), and $-\psi(x)=F_1(\varphi-ux)$, and from Eq. (2.23) we have

$$\left(\frac{\partial}{\partial t}+\vec{e}_z\times\vec{\nabla}(\Omega_1+\varphi)\cdot\vec{\nabla}\right)\times[(\nabla^2-1)\Omega_1+\varphi''(x)+F_1n_0'(x)]=0. \quad (2.27)$$

In a particular case when $f(x)=-F_1n_0''(x)$, Eq. (2.27) can be integrated in the traveling reference frame moving with constant velocity $u$ in the $y$ direction, yielding

$$(\nabla^2-1)\mathcal{O}=\mathcal{K}(\mathcal{O}-ux),\quad \text{where}\quad \mathcal{O}=\Omega_1+\varphi(x). \quad (2.28)$$

Choosing function $\mathcal{K}$ in the form $\mathcal{K}(\mathcal{O}-ux)=C/[\exp(\mathcal{O}-ux)+\exp(-\mathcal{O}+ux)]$ it can be shown [8] that Eq. (2.28) has localized solutions for $\mathcal{O}$ in the form of stationary vortex chains defined by a class of values of the constant $C$, and localized in the $x$ direction.

As in the procedure performed in the preceding subsection, a tripolar vortex solution of Eq. (2.27) can be readily found in a form similar to Eqs. (2.20) and (2.21), by modeling the basic state in the following way: $\varphi(x)=ux-a_1x^2$, and $\varphi''(x)+F_1n_0'(x)=b_1x^2$, which gives the linear shear flow $f(x)=u+2a_1x$, and $n_0(x)=b_1x^3/(3F_1)-2a_1x/F_1$.

## III. CONCLUSION

Two physically very different systems of magnetized and unmagnetized plasmas are studied, and it is found that they are described by rather similar sets of two coupled equations for the magnetic field and temperature perturbations. The equations are derived in the frequency range between the ion and electron plasma frequencies and for spatial scales of the order of the collisionless skin depth, for ions making a neutralizing background, and for negligible density perturbation of electrons. In both cases, some linear types of instabilities (i.e., baroclinic-term-driven and streaming type) are discussed and in the strongly nonlinear limit, for very specific profiles of the basic state functions (the shear flow, the density, the temperature and magnetic field profiles), stationary solutions are found in the form of vortex chains and tripolar vortices.

Equations of the form (2.16) and (2.27) are generic for tripolar vortex solutions; the same sort expressions were obtained earlier in quite different physical problems, viz., in a magnetized dusty plasma [10] and in rotating fluids [11]. The analogy of processes in some very different physical systems is not surprising; an extraordinary similarity is known to exist between problems dealing with nonlinear drift waves in plasmas and Rossby vortices in the atmosphere of our planet [17,18], as well as electron magnetohydrodynamic waves in pulsed magnetic field [19], resulting in the same type of dipolar vortex solutions. However, the presence of the shear flow and nonuniformity of the system, as studied in the present paper, turns out to be of crucial importance for the type of nonlinear tripolar solutions obtained here. Consequently, the fluid-plasma analogy, resulting in tripoles, occurs also in the case of spatially nonuniform systems with sheared flows.

As known from linear theory [3], the inclusion of ion dynamics in the problem introduces new types of modes on a slower temporal scale close to the ion plasma frequency. The nonlinear study of these modes is worth investigating and this work is in progress.

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